ITALIAN-ROMANIAN COLLOQUIUM ON DIFFERENTIAL EQUATIONS AND APPLICATIONS

UDINE, APRIL 10-12, 2019

PROGRAMME

Wednesday

15:00 Registration
15:30 Opening
15:45 J. Mawhin: Homoclinics for some singular strong force Lagrangian systems
16:30 coffee break
17:00 C. Bereanu: Lorentz force equation via Poincaré relativistic Lagrangian
17:45 S. Scrobogna: Asymptotic models for free boundary Darcy flows

Thursday

9:00 E. Rocca: On a sliding mode control for a tumor growth problem
9:45 G. Marinoschi: Sliding mode control of the Hodgkin-Huxley mathematical model
10:30 Posters & coffee
11:00 P. Colli: Optimal control for a phase separation system with a possibly singular potential
11:45 M. Mihailescu: The Monotonicity of the Principal Eigenvalue of the p-Laplace Operator and Related Problems

15:00 P. Omari: On a quasilinear elliptic problem modeling MEMS
15:45 P. Jebelean: Radial Dirichlet systems with Minkowski operator
16:30 Posters & coffee
17:00 R. Precup: A vector linking approach and componentwise solution localization for differential systems
17:45 D. Papini: Periodic solutions to a forced Kepler problem in the plane

Friday

9:00 G. Dal Maso: A maximal dissipation condition for dynamic fracture with an existence result in a constrained case
9:45 C. Cazacu: About some Hardy type inequalities: optimality and minimizers
10:30 coffee break
11:00 C. Serban: Non-radial multiparameter Dirichlet systems with Minkowski operator
11:45 R. Musina: A weighted estimate for generalized harmonic extensions
12:30 Closing

ABSTRACTS

Lorentz force equation via Poincaré relativistic Lagrangian

Cristian Bereanu

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In this talk we explain that the local computation given by Poincaré in his Palermo paper on Special Relativity can be used to obtain periodic solutions for the Lorentz force equation via minimax theorems. This talk is based upon joint works with D. Arcoya, P. Jebelean, J. Mawhin and P. Torres.

About some Hardy type inequalities: optimality and minimizers.

Cristian Cazacu

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In this exposure we discuss both Hardy and Hardy-Rellich type inequalities which establish useful properties for differential operators with singular potentials. We focus on the best constants and the existence/nonexistence of minimizers for potentials with one or various singularities arising either in the interior or on the boundary of a smooth domain. In this concern we also present some preliminary results and formulate some open problems.

This talk is partially supported by a grant of Ministry of Research and Innovation, CNCS-UEFISCDI, project number PN-III-P1-1.1-TE-2016-2233, within PNCDI III.

Optimal control for a phase separation system with a possibly singular potential

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In this talk we discuss a distributed control problem for a phase-field system of conserved type with a possibly singular potential. Two cases are investigated: the viscous Cahn-Hilliard system with a logarithmic-type potential and the standard Cahn-Hilliard system with an everywhere defined regular potential. Necessary first order conditions of optimality are derived under natural assumptions on the data. These results have been obtained in a collaboration with Gianni Gilardi, Gabriela Marinoschi and Elisabetta Rocca, in the framework of an Italian-Romanian research project financed by the Italian CNR and the Romanian Academy.

A maximal dissipation condition for dynamic fracture with an existence result in a constrained case

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We consider a model of elastodynamics with crack growth, based on energy-dissipation balance and a maximal dissipation condition. We prove an existence result in the case of planar elasticity, where the maximal dissipation condition is satisfied among suitably regular competitor cracks. We also examine the case of a kink.

Radial Dirichlet systems with Minkowski operator

Petru Jebelean
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We discuss existence and multiplicity of positive radial solutions to some non-potential Dirichlet systems involving the mean curvature operator in Minkowski space

$$\mathcal{M}(\mathrm{w}) = \mathrm{div}\left(\frac{\nabla \mathrm{w}}{\sqrt{1-|\nabla \mathrm{w}|^2}}\right)$$

in a ball in $\mathbb{R}^N$. Our approach relies on fixed point index theory and on the lower and upper solutions method. Both sublinear and superlinear cases are addressed. The talk is based on joint work with Daniela Gurban.

Sliding mode control of the Hodgkin-Huxley mathematical model

Gabriela Marinoschi
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We present a feedback control design for the action potential of a neuronal membrane in relation with the non-linear dynamics of the Hodgkin- Huxley mathematical model. By using an external current as a control expressed by a relay graph in the equation of the potential, we aim at forcing it to reach a certain manifold in finite time and to slide on it after that. From the mathematical point of view we solve a system involving a parabolic differential inclusion and three nonlinear differential equations via an approximating technique and a fixed point result. The existence of the sliding mode and the determination of the time at which the potential reaches the prescribed manifold is proved by a maximum principle argument. Numerical simulations complete the presentation.

This is a joint work with Cecilia Cavaterra and Denis Enachescu.

Homoclinics for some singular strong force Lagrangian systems

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We study the existence of homoclinic solutions for a class of planar Lagrangian systems

$$\frac{d}{dt}\left(\nabla\Phi(\dot{u}(t))\right) + \nabla_u V(t, u(t)) = 0,$$

where $t \in \mathbb{R}$, $\Phi : \mathbb{R}^2 \to [0, \infty)$ is a G-function in the sense of Trudinger, $V : \mathbb{R} \times \left(\mathbb{R}^2 \setminus \{\xi\}\right) \to \mathbb{R}$ is a C^1-smooth potential with a single well of infinite depth at a point $\xi \in \mathbb{R}^2 \setminus \{0\}$ and a unique strict global maximum 0 at the origin.

Under a strong force condition around the singular point ξ, we prove, via minimization of an action integral and the use of winding number, the existence of at least two geometrically distinct homoclinic solutions $u^{\pm} : \mathbb{R} \to \mathbb{R}^2 \setminus \{\xi\}$. This generalizes a result of Rabinowitz for $\ddot{u} + \nabla_u V(t, u) = 0$.

This is a joint work with Marek Izydorek and Joanna Janczewska.

The Monotonicity of the Principal Eigenvalue of the p-Laplace Operator and Related Problems

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In this talk we present some results obtained in collaboration with Marian Bocea and Julio D. Rossi.

First, we deal with monotonicity with respect to p of the first eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Dirichlet boundary condition. For any fixed integer $D > 1$ we show that there exists $M_D \in [e^{-1}, 1]$ such that for any open, bounded, convex domain $\Omega \subset \mathbb{R}^D$ with smooth boundary for which the maximum of the distance function to the boundary of Ω is less than or equal to M_D, the first eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Dirichlet boundary condition is an increasing function of p on $(1, \infty)$. Moreover, for any real number $s > M_D$ there exists an open, bounded, convex domain $\Omega \subset \mathbb{R}^D$ with smooth boundary which has the maximum of the distance function to the boundary of Ω equal to s such that the principal eigenvalue of the p-Laplacian is not a monotone function of $p \in (1, \infty)$.

Second, we deal with monotonicity with respect to p of the first positive eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Neumann boundary condition. For any fixed integer $D > 1$ we show that there exists $M_N \in [2e^{-1}, 2]$ such that for any open, bounded, convex domain $\Omega \subset \mathbb{R}^D$ with smooth boundary for which the diameter of Ω is less than or equal to M_N, the first positive eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Neumann boundary condition is an increasing function of p on $(1, \infty)$.

Moreover, for each real number $s > M_N$ there exists a sequence of open, bounded, convex domains $\{\Omega_n\}_n \subset \mathbb{R}^D$ with smooth boundaries for which the sequence of the diameters of Ω_n converges to s, as $n \to \infty$, and for each n large enough the first positive eigenvalue of the p-Laplace operator on Ω_n subject to the homogeneous Neumann boundary condition is not a monotone function of p on $(1, \infty)$.

Finally, the minimization problem

$$\Lambda_1(p) := \inf_{u \in X_0 \setminus \{0\}} \frac{\displaystyle\int_\Omega (\exp(|\nabla u|^p) - 1)\, dx}{\displaystyle\int_\Omega (\exp(|u|^p) - 1)\, dx},$$

where $X_0 = W^{1,\infty}(\Omega) \cap (\cap_{q>1} W_0^{1,q}(\Omega))$, is studied when $\Omega \subset \mathbb{R}^D$ $(D \geq 1)$ is an open, bounded, convex domain with smooth boundary and $p \in (1, \infty)$. We show that $\Lambda_1(p)$ is either zero, when the maximum of the distance function to the boundary of Ω is greater than 1, or it is a positive real number, when the maximum of the distance function to the boundary of Ω belongs to the interval $(0, 1]$. In the latter case we provide estimates for $\Lambda_1(p)$ and show that for $p \in (1, \infty)$ sufficiently large $\Lambda_1(p)$ coincides with the first eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Dirichlet boundary condition. Some particular cases and related problems are also discussed.

This presentation is partially supported by CNCS-UEFISCDI Grants No. PN-III-P4-ID-PCE-2016-0035 and No. PN-III-P1-1.1-TE-2016-2233.

A weighted estimate for generalized harmonic extensions

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Motivated by a celebrated paper by Caffarelli and Silvestre [CPDE, 2007], we will discuss some recent results about Hardy and trace-Hardy inequalities for generalized harmonic functions on half-spaces, with applications to symmetry breaking phenomena in some nonlocal problems.

This is a research in collaboration with Alexander I. Nazarov, St.Petersburg Department of Steklov Institute and St.Petersburg State University.

On a quasilinear elliptic problem modeling MEMS

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We investigate existence, multiplicity and qualitative properties of the solutions of the Dirichlet problem for a singularly perturbed quasilinear elliptic equation which appears in the modeling theory of micro-electro-mechanical systems, when the effects of tension and vertical forces are taken into due consideration. This talk is based on joint works with C. Corsato, C. De Coster, F. Obersnel, F. Zoccolan.

Periodic solutions to a forced Kepler problem in the plane

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In this joint work with A. Boscaggin and W. Dambrosio (Dipartimento di Matematica "G. Peano", Università di Torino), we prove that the forced Kepler problem

$$\ddot{x} = -\frac{x}{|x|^3} + \nabla_x U(t,x), \qquad x \in \mathbb{R}^2,$$

has a generalized T-periodic solution, according to the definition given in [1]. Here $U : \mathbb{R}\times\mathbb{R}^2$ is a smooth function, T-periodic in the first variable and satisfying $U(t,x) = \mathrm{O}(|x|^\alpha)$ for some $\alpha \in (0,2)$ as $|x| \to \infty$. The proof relies on variational arguments.

References

[1] Boscaggin, Ortega, Zhao: Periodic solutions and regularization of a Kepler problem with time-dependent perturbation. *Trans. Amer. Math. Soc.* (2018).

A vector linking approach and componentwise solution localization for differential systems

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A vector notion of linking is introduced in order to obtain, via a minimax principle, the existence and componentwise localization of positive solutions to differential systems of gradient type. The new approach allows the energy functional to combine minimization properties with respect to a part of the variables, and the mountain pass geometry for the other variables.

References

[1] R. Precup: Critical point theorems in cones and multiple positive solutions of elliptic problems, *Nonlinear Anal.* **75** (2012), 834–851.
[2] R. Precup: A variational analogue of Krasnoselskii's cone fixed point theory, in *Proceedings of the International Conference in Nonlinear Analysis and Boundary Value Problems, Santiago de Compostela, 2018*, Springer, to appear.
[3] R. Precup, P. Pucci and C. Varga: Energy-based localization and multiplicity of radially symmetric states for the stationary p-Laplace diffusion, *Complex Var. Elliptic Equ.*, to appear.

On a sliding mode control for a tumor growth problem

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In this talk we present a joint work with P. Colli, G. Gilardi and G. Marinoschi. We study the sliding mode control (SMC) problem for a diffuse interface tumor growth model coupling a viscous Cahn-Hilliard type equation for the phase variable with a reaction-diffusion equation

for the nutrient. First, we prove the well-posedness and some regularity results for the state system modified by the state-feedback control law. Then, we show that the chosen SMC law forces the system to reach within finite time the sliding manifold (that we chose in order that the tumor phase remains constant in time). The feedback control law is added in the Cahn-Hilliard type equation and leads the phase onto a prescribed target in finite time.

Asymptotic models for free boundary Darcy flows

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In the first part of the talk I will provide rigorous asymptotic models for the free boundary Darcy problem under the assumption of weak nonlinear interaction, in a regime in which the steepness parameter of the interface is considered to be very small. The second part of the talk will be devoted to prove some rigorous result of well-posedness in critical spaces for the asymptotic model. Joint work with R. Granero-Belinchn.

Non-radial multiparameter Dirichlet systems with Minkowski operator

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We study a (λ_1, λ_2)-parameterized Dirichlet system with mean curvature operator in Minkowski space

$$\mathcal{M}(u) = \operatorname{div}\left(\frac{\nabla u}{\sqrt{1-|\nabla u|^2}}\right),$$

in a general bounded domain in $\mathbb{R}^N$ and allowing, among others, nonlinearities of Lane-Emden type. For such a system we provide sufficient conditions ensuring the existence of a hyperbola like curve which separates the first quadrant in two disjoint sets, an open one $\mathcal{O}$ and a closed one $\mathcal{F}$, such that the system has zero or at least one strictly positive solution, according to $(\lambda_1, \lambda_2) \in \mathcal{O}$ or $(\lambda_1, \lambda_2) \in \mathcal{F}$. Moreover, we show that inside of $\mathcal{F}$ there exists an infinite rectangle in which the parameters being, the system has at least two strictly positive solutions. Our approach relies on a lower and upper solutions method - which we develop here, together with topological degree type arguments. In a sense, our results extend to non-radial systems some recent existence/non-existence and multiplicity results obtained in the radial case.

The talk is based on joint work with D. Gurban and P. Jebelean.

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